International Journal of Humanoid Robotics
Vol. 2, No. 1 (2005) 105–124

ROBOTIC HANDWRITING

VELJKO POTKONJAK

Faculty of Electrical Engineering, University of Belgrade
Bulevar kralja Aleksandra 73, 11000 Belgrade, Serbia and Montenegro
potkonjak@yahoo.com

Received 28 November 2003
Revised 6 April 2004
Accepted 19 April 2004

Handwriting has always been considered an important human task, and accordingly it has attracted the attention of researchers working in biomechanics, physiology, and related fields. There exist a number of studies on this area. This paper considers the human–machine analogy and relates robots with handwriting. The work is two-fold: it improves the knowledge in biomechanics of handwriting, and introduces some new concepts in robot control. The idea is to find the biomechanical principles humans apply when resolving kinematic redundancy, express the principles by means of appropriate mathematical models, and then implement them in robots. This is a step forward in the generation of human-like motion of robots. Two approaches to redundancy resolution are described: (i) "Distributed Positioning" (DP) which is based on a model to represent arm motion in the absence of fatigue, and (ii) the "Robot Fatigue" approach, where robot movements similar to the movements of a human arm under muscle fatigue are generated. Both approaches are applied to a redundant anthropomorphic robot arm performing handwriting. The simulation study includes the issues of legibility and inclination of handwriting. The results demonstrate the suitability and effectiveness of both approaches.

Keywords: Handwriting; man–machine analogy; human-like motion; redundancy resolution; robot fatigue; humanoid.

1. Introduction

Humanoid robots have recently finally been recognized as the main direction in the entire study of robotics.[1] One may say that the idea of robots came from a human desire to create a copy of itself. This "childish" need was followed by the work of craftsmen and engineers who, step by step, starting from toys and dolls, developed anthropomorphic devices which were technologically sophisticated, scientifically based, and even applicable.[2–4] These walking machines represented the true start of robotic science. The industrial potential of robots turned the focus of research to practical problems of automation. It took researchers and manufacturers a long time to solve so many different problems in industry. The accumulated knowledge and experience, the technology growth, and the saturation in industrial robotics, has allowed the robotic community to recognize the service robots and especially humanoid robots as a new and profitable direction of work. Recent results show that this was a prospective idea.

Since humanoid robots are seen to be applicable in services, house work and other activities that require close cooperation with humans, it was necessary to supply them with the ability to move in a human-like fashion, to communicate in a human-like manner, and to feature human-like intelligence.[1] The first ability, being the topic of this article, has required an extensive study of biomechanics and the human–robot analogy. This is how we came to handwriting — a task that seemed to be appropriate only for humans. So, we now pose a crucial question: *what do robots have to do with handwriting*? There are a few answers. Handwriting, being a typical human motion, is a highly demanding task regarding kinematics and dynamics. It involves a redundant number of joints (degrees of freedom — DOF). So, handwriting is seen as a "perfect test" for humanoids and even industrial robots. The other aspect concerns the possibility to improve robot control by learning from humans. Human handwriting engages different levels of motion control: learned patterns (with all the associated problems), on-line tracking, etc. By studying the biomechanics of handwriting, one can learn about control concepts, skill acquisition, redundancy resolution, etc. So, perhaps robots will never have to write by hand, but the study of this possibility is still very useful. However, the word "never" should be used conditionally — if humanoids continue to improve their human-likeness, true robot handwriting might become reality.

2. Handwriting: From Human to Robot

Handwriting is considered an important human task, and accordingly it has attracted the attention of researchers working in biomechanics, physiology, and related fields. There exist a number of studies on this area. Since the majority of them are not of direct interest to our work in robotics, we simply refer to the website *www.psychomot.ups-tlse.fr/Ecriture.rtf*, where an extensive listing of such studies may be found, and to Ref. 5, where relevant biomechanics results are explained. The work of Potkonjak and his associates[5–7] was the first to relate robots with handwriting. The work was two-fold: it improved the knowledge in biomechanics of handwriting, and introduced some new concepts in robot control. The idea was to find the biomechanical principles humans apply when resolving redundancy, and to implement these principles in robots. The robotic background for this work was found in the concept of micro–macro manipulation.[8]

In Refs. 9–12, the concept of distributed positioning (DP) was proposed to resolve redundancy and improve robot kinematic and dynamic performance. It suggested separation of required motion into a smooth global and fast local motion. These components should be distributed to a redundant number of joints in accordance with their inertial properties: high-inertia joints should take care of smooth global motion while low-inertia redundancy is engaged to solve highly accelerated local motion. The idea was to enable massive industrial robots to perform fast and precise manipulation. References 6 and 7 introduced handwriting as a test-motion for checking the efficiency of the DP concept. Reference 5 considered an anthropomorphic arm engaged in handwriting. Due to a higher degree of redundancy, the DP concept could not resolve it completely. The pseudoinverse (optimization) was needed to solve the wrist motion. The obtained results related some important characteristics of handwriting: legibility, inclination of letters, and engagement of fingers (fingers were critical due to relatively quick fatiguing). It was shown that for a given level of legibility, there existed an optimal inclination that minimized the engagement of fingers.

Lately, the human–robot analogy has led to the study of the behavior of a "fatigued robot." The reason for this was the fact that humans use their redundancy to avoid, or at least delay, fatigue problems. When feeling fatigue in some joint, a human reconfigures itself; by engaging other joints more, the exhausted joint is given a chance to rest. This reconfiguration does not compromise the task execution. The idea was to apply the same principle to robots when overloaded. The next benefit from research in fatigue problems is the possibility of achieving some human-like communication. The mentioned reconfiguration, which takes place with fatigued humans, can be observed and it represents a message sent to people in the surroundings. We wish the robot to behave in the same manner so that we can recognize when it is overloaded. These problems have been elaborated in Refs. 13–18. The biological background — a description of fatigued muscle behavior — can be found in Refs. 19 and 20.

3. Robot Arm Kinematics and Dynamics

A robot arm with n DOF is described by means of n joint coordinates (internal or configuration coordinates) forming the configuration vector $\mathbf{q} = [q_1, \ldots, q_n]^T$. From the task point of view, one is concerned with the end-effector motion described by means of external coordinates $\mathbf{X} = [x\, y\, z\, \theta\, \varphi\, \psi]^T$, where x, y, and z are Cartesian coordinates, and θ, ϕ, and ψ are orientation angles (yaw, pitch and roll, respectively). Another important concept is the operational space. It is a subset of external positions, containing those external coordinates responsible for the execution of a given task. Let the operational vector be denoted by $\mathbf{x}$ and have the dimension of $m \leq 6$.

A kinematic model understands the relation between configuration and operational space. In its first order and second order forms, the model is

$$\dot{\mathbf{x}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}}, \quad \ddot{\mathbf{x}} = \mathbf{J}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{K}(\mathbf{q},\dot{\mathbf{q}}), \tag{1}$$

where $\mathbf{J} = \partial\mathbf{x}/\partial\mathbf{q}$ is the Jacobian matrix of dimension $m \times n$ and $\mathbf{K} = \partial^2\mathbf{x}/\partial\mathbf{q}^2\dot{\mathbf{q}}^2$ is the $m \times 1$ adjoint vector. Redundancy resolution requires so-called inverse kinematics, i.e. the calculation of $\mathbf{q}$ for a given $\mathbf{x}$. If $m = n$, the system is non-redundant and a unique solution is possible. If $m < n$, the system is redundant and there exists an infinite number of solutions of the inverse kinematics, meaning that different configuration motions can produce the same operational motion. If one solution is to be selected, then additional requirements, which will "employ" the redundancy, have to be imposed. A redundant arm usually has a heavy part consisting of m joints, which is called the "nonredundant basic configuration." The rest of the arm ($n-m$ joints) constitutes the redundancy.

The dynamics of the arm — the mechanical part plus second order actuators — is described by the well-known model:

$$\hat{\mathbf{H}}(\mathbf{q})\ddot{\mathbf{q}} + \hat{\mathbf{h}}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{u}, \tag{2}$$

where $\mathbf{u}$ is the vector of the control inputs, $\hat{\mathbf{H}}$ is the inertial matrix, and $\hat{\mathbf{h}}$ takes care of velocity-dependent effects. The dynamic model is used to simulate the system behavior.

4. Modeling Handwriting Synergy — DP Concept

4.1. *Principles and mathematics*

DP is formulated by analogy with human behavior and is intended to model a robot arm involved in fast manipulation. The required robot task, end-effector operational motion $\mathbf{x}(t)$ of dimension m, is assumed to have m_a highly accelerated elements. These elements form the subvector $\mathbf{x}_1(t)$. The other elements are smooth [subvector $\mathbf{x}_2(t)$]. Now, $\mathbf{x} = [\mathbf{x}_1, \mathbf{x}_2]^T$. We suppose a situation where the massive basic non-redundant configuration (vector $\mathbf{q}_b$ of dimension m) cannot solve the task due to the presence of accelerations. The DP concept resolves this problem.

The basic non-redundant configuration (m-dimensional $\mathbf{q}_b$) is supplemented by a low-inertia redundancy ($\mathbf{q}_r$ of dimension n_r). The entire configuration is now $\mathbf{q} = [\mathbf{q}_b, \mathbf{q}_r]^T$, and has a dimension of $n = m + n_r$.

Accelerated motion $\mathbf{x}_1(t)$ is separated in two components: a smooth component $\bar{\mathbf{x}}_1(t)$ and a highly accelerated component $\tilde{\mathbf{x}}_1(t)$; thus $\mathbf{x}_1 = \bar{\mathbf{x}}_1 + \tilde{\mathbf{x}}_1$. Some suitable smoothing method is to be applied (a low-pass filter could be used to make this separation). The "basic operational motion" is now defined to be the motion that contains the smoothed component $\bar{\mathbf{x}}_1(t)$ and the subvector $\mathbf{x}_2$ (being already smooth): $\mathbf{x}_b = [\bar{\mathbf{x}}_1, \mathbf{x}_2]^T$. The basic non-redundant configuration $\mathbf{q}_b$ is capable of solving the motion $\mathbf{x}_b$. Mathematically, the solution for $\mathbf{q}_b(t)$ involves the inverse of a non-redundant quadratic ($m \times m$) Jacobian. This represents the *first step* in the DP concept.

The redundancy is now forced to solve the high accelerations $\tilde{\mathbf{x}}_1(t)$. The necessary condition (regarding dimensions) is $n_r \geq m_a$. In the original concept, equality held ($n_r = m_a$) and the unique solution for $\mathbf{q}_r(t)$ was obtained.[9–12] This constituted the *second step* in resolving the inverse kinematics. Thus, the entire configuration motion $\mathbf{q}(t)$ was found. Besides industrial tasks, the concept was checked for the handwriting example.[6,7] The idea for a handwriting test-task follows the fact that letters require high accelerations and a human solves them by distributing the pencil motion between the massive arm and the low-inertia fingers.

When the focus was moved from industrial robots to humans and humanoids, it was recognized that the wrist joint played an essential role in handwriting. The wrist allows long-term fast writing by reducing the involvement of fingers that are precise but fatigue quickly. The wrist is responsible for the inclination of letters, often present with humans. The introduction of the wrist increases the entire degree of redundancy, causing $n_r > m_a$. The *second step* now cannot be performed as described above. The *first step* reduces the redundancy degree from n_r to $n_r - m_a$ but does not eliminate it completely. So, the *second step* needs an additional condition and it is always some optimality criterion. Among different options presented in the literature, we select minimization of finger involvement. This comes from the fact that fingers can move very precisely but cannot stand long-term fast movement. To measure the finger involvement, an integral criterion has been suggested[5]: IKI — the integral kinematic involvement, being the sum of amplitudes of fingers motions. Some other reasonable criteria (reducing energy or motor temperatures) produced results rather comparable with IKI ones.[18]

4.2. *Example*

In a simplified (but still representative) example we consider a planar arm consisting of the shoulder q_1, the elbow q_2, and the wrist q_3 (Fig. 1). In writing, the fingers work together to produce two translations as shown in Fig. 2. Hence, with the robot arm, true fingers are substituted by two linear joints (q_4 and q_5 in Fig. 1). The motion ranges for such "sliding fingers" are $\Delta_4 = q_{4\,\max} - q_{4\,\min} = 0.05\,\mathrm{m}$ and $\Delta_5 = q_{5\,\max} - q_{5\,\min} = 0.05\,\mathrm{m}$. The complete set of parameters used in the example is given in the Appendix.

The task consists of writing a prescribed sequence of letters shown by solid lines in Fig. 3. Under (a), an x–y representation is presented (x and y being operational coordinates), while (b) and (c) show the time histories $x(t)$ and $y(t)$. This reference sequence is set so as to be close to real letters and, at the same time, to be easy to describe mathematically (cycloids, circles, and straight lines have been used).

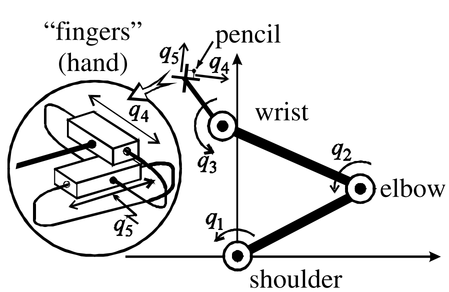

Fig. 1. Mechanism configuration with five DOF.

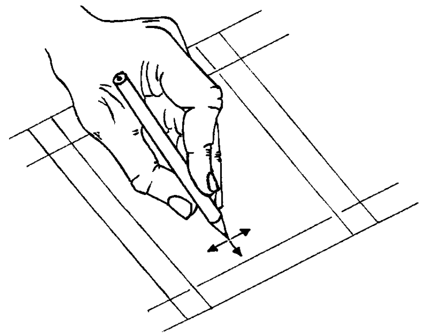

Fig. 2. Coordinated motion of fingers produces two translations.

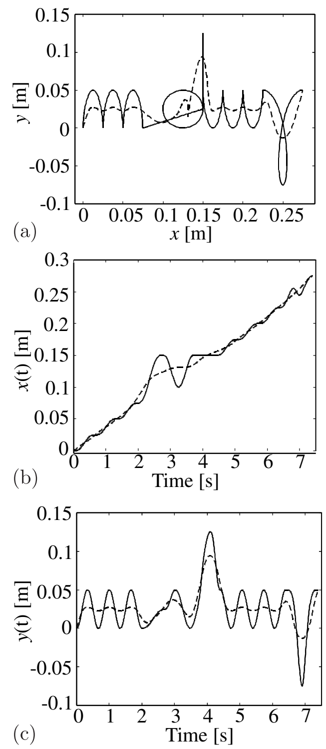

Fig. 3. Definition of the task: (a) sequence of letters; (b), (c) time histories of operational coordinates. The solid lines show the prescribed motion, and the dashed lines show the smooth components.

For this example, it holds that $\mathbf{x} = \mathbf{x}_1 = [x, y]^T$, $m = m_a = 2$, and $\mathbf{x}_2$ does not exist. Smoothing requires separation: $x = \bar{x}+\tilde{x}$, $y = \bar{y}+\tilde{y}$. The results, smooth components $\bar{x}(t)$ and $\bar{y}(t)$, are indicated in Fig. 3 by dashed lines. The basic operational motion contains these smooth components: $\mathbf{x}_b = \bar{\mathbf{x}}_1 = [\bar{x}, \bar{y}]^T$.

The mechanism configuration is separated into two functional parts. The shoulder and elbow constitute the non-redundant basic configuration: $\mathbf{q}_b = [q_1, q_2]^T$, $m = 2$. The wrist and the two linear "fingers" represent the redundancy: $\mathbf{q}_r = [q_3, q_4, q_5]^T$, $n_r = 3$.

The basic configuration $\mathbf{q}_b$ cannot handle the original task $\mathbf{x}(t) = [x(t), y(t)]^T$ due to the presence of high accelerations. The *first step* of the DP concept is to force the basic configuration to solve the smooth motion $\mathbf{x}_b = [\bar{x}, \bar{y}]^T$. In order to get the maximum from the configuration, minimum smoothing (by using the "sliding window" method) is performed, i.e. just to the level that the configuration can handle.

Once the motion $\mathbf{q}_b(t)$ is found, we start the *second step* in order to solve for the redundancy $\mathbf{q}_r$. Since $n_r = 3 > m_a = 2$, the *second step* still faces the problem of redundant DOF: two operational motions, $\tilde{\mathbf{x}}_1 = [\tilde{x}, \tilde{y}]^T$, are to be solved by using three configuration coordinates, $\mathbf{q}_r = [q_3, q_4, q_5]^T$. In order to get a unique solution of the inverse kinematics, we introduce an optimality criterion by trying to minimize the involvement of fingers. The IKI criterion is applied. The solid lines in Fig. 4 show the results, time histories $\mathbf{q}_r(t)$. One can observe that the motions of fingers (q_4 and q_5) violate the ranges Δ_4 and Δ_5 (while writing letters "d" and "j"), meaning that the solution found cannot be realized. Any attempt to do this would lead to the incorrect shape of letters and reduced legibility. This is a consequence of the fact that the wrist is not of great help when letters are strictly vertical, and accordingly, too much is required from the fingers (they are not long enough). In order to allow the wrist to help more efficiently, we modify the task (i.e. the reference) by inclining the letters. Example of inclined writing (for the angle $\alpha = 30°$) is shown in Fig. 5. With the inclination, the engagement of the wrist (q_3) increases and the engagement of the fingers (q_4 and q_5) reduces. This is obvious from Fig. 4 (different kinds of line are used for different inclination). After an inclination of 24°, translation q_4 falls into the allowable region Δ_4, while, after 34°, the other translation q_5 falls into Δ_5. This means that any sequence, inclined at 34° or more, can be written "ideally."

After introducing inclination, we make a step forward and note a general fact that humans often do not insist on the ideal execution of a given task. In the current example, handwriting, this means that deformed letters are acceptable if still legible. This relaxed condition opens the possibility for some additional optimization. Here, we prescribe some level of legibility and try to further reduce the involvement of fingers (IKI criterion). The legibility of a sequence of letters is defined on the basis of the mean square deviation from the ideal sequence. If e is the mean square error, then legibility is its normalized value, $L_e = (e_{\max} - e)/(e_{\max} - e_{\min})$, being in the interval $L_e \in [0, 1]$. For the ideal sequence (i.e. the reference), it holds that $e = e_{\min} = 0$ and $L_e = 1$. The values $e = e_{\max} = 0.0163$ and $L_e = 0$ stand for the threshold — the lowest legibility still worth considering. Let us note that Ref. 5 used a modified definition based on a function that introduced a subjective feeling of legibility.

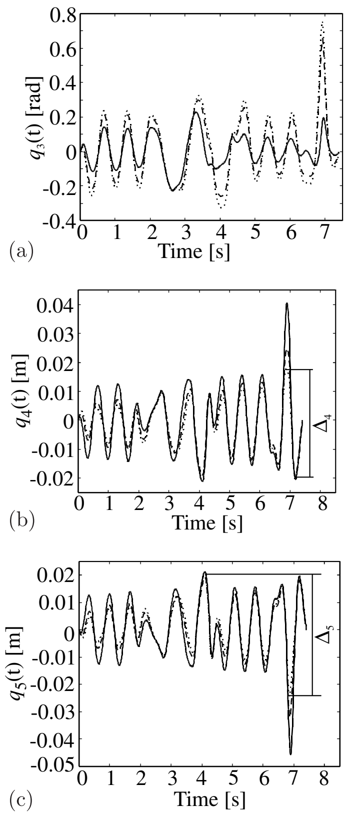

Fig. 4. Solution to the motion of the redundancy (the solid lines stand for strictly vertical letters, the dashed lines for inclined $\alpha = 24°$ writing, and the dotted lines for the inclination of $\alpha = 34°$).

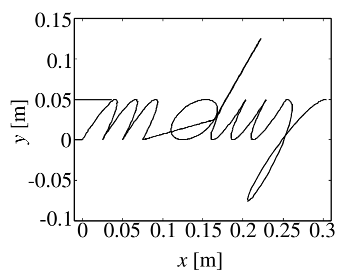

Fig. 5. Inclined letters ($\alpha = 30°$).

Figure 6 presents the results; it relates the involvement of fingers (IKI), inclination (α), and legibility (L_e). Each curve corresponds to some level of legibility L_e and shows how IKI depends on the inclination angle α. Each curve features a clear minimum. Reducing the legibility, the point of minimum moves slightly to the left

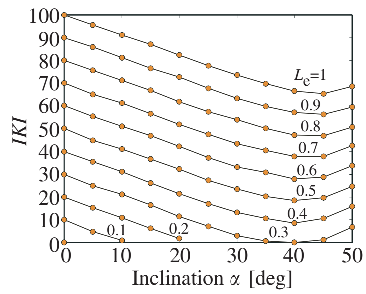

Fig. 6. Relation of finger involvement (IKI), inclination of writing (α), and legibility (L_e).

(towards smaller inclinations). Observing the diagram, one can conclude that, for any selected level of legibility, there exists an optimal inclination that minimizes the involvement (IKI) of fingers (for instance, if legibility is prescribed to be 0.6, the optimal inclination is 40°, and the corresponding IKI is about 28). For other criteria (IKI replaced by energy consumption or by motor heating) the diagrams feature similar behavior.[18]

4.3. *Discussion on application*

The DP concept shows itself to be a good model to describe human motion in handwriting. Humans really distribute the prescribed motion to a redundant number of joints in accordance with their inertial characteristics and the muscle potentials. On the other hand, the method is very suitable for implementing in robots. An interesting issue left for discussion, is how to smooth the accelerated motion. With humans, this is a question of learning. The same principle may be introduced for robots but it opens a complex problem of machine learning. It is expected that smoothing may be successfully solved by using some appropriate low-pass filter (as was done above). One, however, notes that for on-line applications, the adaptation of the cut-off frequency is a problem that deserves separate treatment.

5. Robot Fatigue — A New Option in Human–Robot Communication

5.1. *Principles*

If the human arm is given long-term or heavy work, fatigue will appear. Until the symptoms of fatigue appear, we talk about *Regular Motion*. When the fatigue in some muscles of the human arm exceeds the threshold level, the arm tends to reconfigure itself and thus disturbs the steady state imposed by the DP concept. On-line *Reconfiguration* is needed since it must be based on the current level of fatigue. Reconfiguration means depressed involvement of the exhausted joint and a higher engagement of the others. In this way, the exhausted joint (or joints) is (are) given a chance to rest. This reconfiguration is an "inner" process, meaning that it does not effect the correct execution of the task. Mathematically speaking, a redundant system has an infinite number of configuration motions for one operational motion, and reconfiguration means the selection of a new configuration from this set.

When a fatigued human changes posture, this can be observed, and thus, reconfiguration represents a message about his state. People in the surroundings may react to the message although the task execution is not compromised.

If the heavy-duty task lasts too long, then arm joints, one by one, will become fatigued. After a few reconfigurations, there will be no joint able to help. From this moment, task execution will no longer be correct. Deviations will appear and we talk about the *Degeneration* phase. This can be considered a new message to the surroundings.

Here, we try to model this behavior and apply it to the robot arm. A measure of robot joint fatigue is the motor temperature. The threshold is the temperature that exceeds the allowable level, meaning that the arm is overloaded.

5.2. *Mathematics*

Regular Motion. Redundancy resolution is based upon the DP concept, along with the request for the maximal comfort. This follows from the observed behavior of humans.[21] Instead of a low-pass filter used for DP in Sec. 4, here we directly apply the method of pseudoinverse. To achieve this, the appropriate criterion is introduced:

$$\Omega(\dot{\mathbf{q}}) = 0.5 \cdot \dot{\mathbf{q}}^T \mathbf{W}' \dot{\mathbf{q}} + 0.5 \cdot (\dot{\mathbf{q}} - \dot{\mathbf{q}}_\alpha)^T \mathbf{W}'' (\dot{\mathbf{q}} - \dot{\mathbf{q}}_\alpha), \tag{3}$$

where $\mathbf{W}'$ and $\mathbf{W}''$ are $n \times n$ positive-definite symmetric weighing matrices. The first term enables penalization of the motion of some joints relative to others and is used to distribute the joint motions in accordance with the DP concept (i.e. to stimulate the motion of low-inertia joints and penalize the motion of high-inertia joints). By changing the weighing matrix $\mathbf{W}'$, this term enables a proper reconfiguration of the robot in accordance with the actual progress of fatigue. The second term is used to maximize the comfort. The comfortable motion of a joint is seen as the motion being near the middle position of the joint range. According to Ref. 22, $\dot{\mathbf{q}}_\alpha = -k_\alpha(\partial G(\mathbf{q})/\partial \mathbf{q})^T$, and to maximize the comfort we define $G(\mathbf{q})$ as a measure of deviation from the middle values.[21,23]

The minimization of the criterion (3) is performed via the method of Lagrange multipliers. The Lagrangian corresponds to the functional (3) and the kinematic constraint (1). The calculation of the configuration velocities $\dot{\mathbf{q}}$ involves the weighed pseudoinverse of the Jacobian matrix (according to Ref. 24):

$$\dot{\mathbf{q}}^* = \mathbf{J}_W^{\#} \dot{\mathbf{X}}^* + \left(\mathbf{I} - \mathbf{J}_W^{\#} \mathbf{J}\right) \mathbf{W}^{-1} W'' \dot{\mathbf{q}}_\alpha, \tag{4}$$

where $\mathbf{W} = \mathbf{W}' + \mathbf{W}''$, and $\mathbf{J}_W^{\#} = \mathbf{W}^{-1}\mathbf{J}^T(\mathbf{J}\mathbf{W}^{-1}\mathbf{J}^T)^{-1}$ is the weighted pseudoinverse of the Jacobian matrix. The sign * is used to indicate that the reference motion is in question.

Regarding the control, a PD regulator could be adopted: $u_j = K_{Pj}(q_j^* - q_j) + K_{Vj}(\dot{q}_j^* - \dot{q}_j)$, $j = 1, \ldots, n$, where q_j^* is the reference position, q_j is the actual position, K_{Pj} and K_{Vj} are position and velocity feedback gains. Applying this control law, the motion $q_j(t)$, $j = 1, \ldots, n$ will result, which is expected to track closely the reference $q_j^*(t)$.

Reconfiguration. We now look for a mathematical method to force reconfiguration in accordance with the actual progress of fatigue. With robots, motor temperature Θ_j is the measure of fatigue in joint j. The critical value $\Theta_{j,cr}$ is defined for each motor, limiting the desired motor working mode. It is not a final limit but rather a bound of a desirable region. Above the critical value, the robot joint "feels fatigue." Some appropriate algorithm should force the redistribution of engagement in order to relax the exhausted joint. The algorithm introduces penalty functions into the weighing matrix:

$$\mathbf{W} = \mathrm{diag}[\varphi_1(\Theta_1), \ldots, \varphi_n(\Theta_n)]. \tag{5}$$

"Penalty functions" $\varphi_j(\Theta_j)$ should penalize the exhausted joints and stimulate those that are still "fresh." Mathematically speaking, $\varphi_j(\Theta_j)$ should be constant until Θ_j reaches $\Theta_{j,cr}$, and monotonically increasing above $\Theta_{j,cr}$. In this way, the penalty functions $\varphi_j(\Theta_j)$ will contribute to reduced movement of each joint in which the actual value of fatigue exceeds an assigned critical limit. The choice of a particular penalty function is task dependent. One possibility, used in the simulation study of this article, is a quadratic function:

$$\varphi_j(\Theta_j) = \begin{cases} w_j, & \Theta_j < \Theta_{j,cr}, \\ w_j + k_{\varphi,j}(\Theta_j - \Theta_{j,cr})^2, & \Theta_j \geq \Theta_{j,cr}, \end{cases} \tag{6}$$

where the initial weighting factor w_j is a scalar constant, and the coefficient $k_{\varphi,j} > 0$ determines the desired slope of the penalty function.

When feeling fatigue in some joints, the robot will reconfigure itself in the above way. This is done while keeping the required operational trajectory (thus, reconfiguration does not effect the execution of the task). It is expected that the reduced engagement of exhausted joints will give them a chance to rest and go out of the critical working mode. Several reconfigurations may happen, one after the other, as different joints reach the critical levels. If the task is not too tough, the robot will finally find a steady state in which it can operate for a longer time (some results that support this expectation are reported in Refs. 13–15). To control the robot we still use the PD regulator.

Degeneration. If the task imposed on the robot is too demanding, it may happen that, in spite of reconfiguration, the motor temperatures continue to rise. This means that the reconfiguration will delay the fatigue problem but will not eliminate it. To handle this situation, some upper limits for the temperatures are adopted, i.e. $\Theta_{j,\max}$, $j = 1, \ldots, n$. These limits indicate the point of entering a dangerous motor working mode. In this situation, a further rise in temperature must be prevented regardless of the quality of the output work. This is done by activating a "current limiter." Limiting motor current, being the source of heating, should stop the rise of temperature. The limiter will allow the current that is smaller than the required value by the factor D, and thus, for joint j it will be

$$i_j = D_j(\Theta_j) i_j^{\text{req}}, \tag{7}$$

where i_j is the actual current and i_j^{req} is the value required by the dynamics of the given task. $D_j(\Theta_j)$ is called the "current-damping factor." It depends on the actual level of temperature (fatigue). In order to efficiently relax the joint in accordance with its fatigue, a decreasing function is adopted:

$$D_j(\Theta_j) = \begin{cases} 1, & \Theta_j \leq \Theta_{j,\max}, \\ e^{-(\Theta_j - \Theta_{j,\max})}, & \Theta_j > \Theta_{j,\max}. \end{cases} \tag{8}$$

Damping the current will result in insufficient joint torques and accordingly in the degeneration of the actual trajectory. The reference configuration motion $\mathbf{q}^*(t)$ will still emerge from the imposed (reference) task trajectory $\mathbf{x}^*(t)$, but the limited joint torques will result in actual motion $\mathbf{q}(t)$ that might be far away from the reference. As a result, the actual task trajectory $\mathbf{x}(t)$ will be considerably degenerated.

Thus, in the third phase, the robot will still "try to do the job," but since "it is tired," the results will be unsatisfactory.

For simulation purposes we need a mathematical model that relates the source of thermal energy (i.e. rotor winding current) and the temperatures of the rotor and the housing.[13,14] The thermal dynamics model involves the thermal capacities of the rotor and the housing and the transfer of energy, rotor-to-housing and housing-to-ambient. The second order model (for the jth joint motor) is

$$T_{rj}\dot{\Theta}_{rj} = Z_{rj} \cdot R_j i_j^2 - (\Theta_{rj} - \Theta_{hj}), \quad T_{hj}\dot{\Theta}_{hj} = \frac{Z_{hj}}{Z_{rj}} f(\Theta_{rj} - \Theta_{hj}) - (\Theta_{hj} - \Theta_a), \tag{9}$$

where Θ_{rj} and Θ_{hj} are the rotor and housing temperatures, T_{rj} and T_{hj} are the thermal time constants, Z_{rj} and Z_{hj} are the energy-transfer resistances rotor-to-housing and housing-to-ambient, Θ_a is the ambient temperature, and $R_j i_j^2$ represents the Joule power loss. The time constants influence the slope of the temperature progress while the resistances define the steady state levels. The thermal dynamic model can be reduced to first order if the appropriate choice of parameters is made. All the relevant effects will be preserved.[15–17] The first order model is

$$T_j \Theta_j = Z_j R_j i_j^2 - (\Theta_j - \Theta_a). \tag{10}$$

The thermal model, along with the dynamic model of the arm [Eq. (2)], enables simulation.

5.3. *Example*

We consider the robotic arm shown in Fig. 1 in Sec. 4. The task (i.e. the reference) in that example was defined to be flexible, allowing different inclinations of letters. For the present analysis, we set the inclination to $\alpha = 20°$ [as can be seen in Fig. 10(a)].

Simulation in this work is performed to prove the feasibility of the concept. Thus, the system parameters need not be realistic but rather chosen so as to stress the relevant effects. In addition, a overly long simulation should be avoided. Starting from this, we adopted the appropriate values for the system parameters. The complete set of parameters used in the example is given in the appendix.

To show the most interesting simulation effects, we will explore the behavior of joints 4 and 5 ("fingers"), and the overall execution of the task.

Figure 7 shows the behavior of joint 4. Figure 7(a) presents the progress of motor temperature (joint fatigue Θ_4). Figure 7(b) presents the variation of joint involvement. As a measure showing how much a particular joint (e.g. the jth one) is involved in the task execution, a variable called "kinematic involvement" — KI_j — is introduced. This is calculated for each repetition of the sequence of letters: $KI_j = \int_{T=9s} |\dot{q}_j|\, dt$, where $T = 9\,\text{s}$ is the time needed to accomplish one sequence. Figure 7(c) shows the reference motion of the joint, $q_4^*(t)$, and Fig. 7(d) shows its real motion ($q_4(t)$). Figure 8 presents the behavior of joint 5.

Figure 9 shows the error in the task execution. This is the deviation (DEV) from the ideal sequence of letters, i.e. from the reference trajectory (x^*, y^*). The error is calculated for each repetition of the sequence and represents the normalized mean square error over the sequence.

Let us discuss the simulation results.

Phase 1 — Regular Motion lasts for $t \in [0, t_1 \approx 80\,\text{s}]$. Phase 1 starts immediately and lasts until the fatigue in some joint (motor temperature Θ_j) exceeds the assigned critical level $\Theta_{j,cr}$. In this phase the continuous progress of fatigue in both joints (4 and 5) is monitored (Figs. 7(a) and 8(a)). The joint involvements are at a constant level [Figs. 7(b) and 8(b)] meaning a steady situation in the distribution of the task to robot joints. This steady distribution is supported by Figs. 7(c) and (d) and 8(c) and (d), where the oscillations with constant magnitudes can be observed. In this phase, the error of writing (DEV in Fig. 9) is rather small. Phase 1 ends at about $t_1 = 80\,\text{s}$ when joint 5 feels fatigue, i.e. the motor temperature exceeds the critical level: $\Theta_5 \geq \Theta_{5,cr}$ [see Fig. 8(a)].

Phase 2 — Reconfiguration lasts for $t \in [t_1 \approx 80\,\text{s}, t_2 \approx 190\,\text{s}]$. When joint 5 feels fatigue, phase 2 begins. Reconfiguration starts since the penalty function in joint 5 forces its reduced engagement. This reduction appears as a drop in the involvement KI_5 at $t_1 = 80\,\text{s}$ [Fig. 8(b)]. This is also obvious in Figs. 8(c) and (d), where the magnitudes of oscillations decrease. Since the other joints have to help, one may observe the increased involvement KI_4 [Fig. 7(b)]. This higher engagement of joint 4 can be recognized in Figs. 7(c) and (d) as an increased density of the oscillation diagrams. Joint 4 is not the only one to help. So, if the behavior of joint 3 was depicted, it would feature increased involvement as well.

During phase 2, at about $t' = 160\,\text{s}$, the temperature in joint 4 reaches the critical level: $\Theta_4 \geq \Theta_{4,cr}$ [see Fig. 7(a)]. At that moment, the penalty function

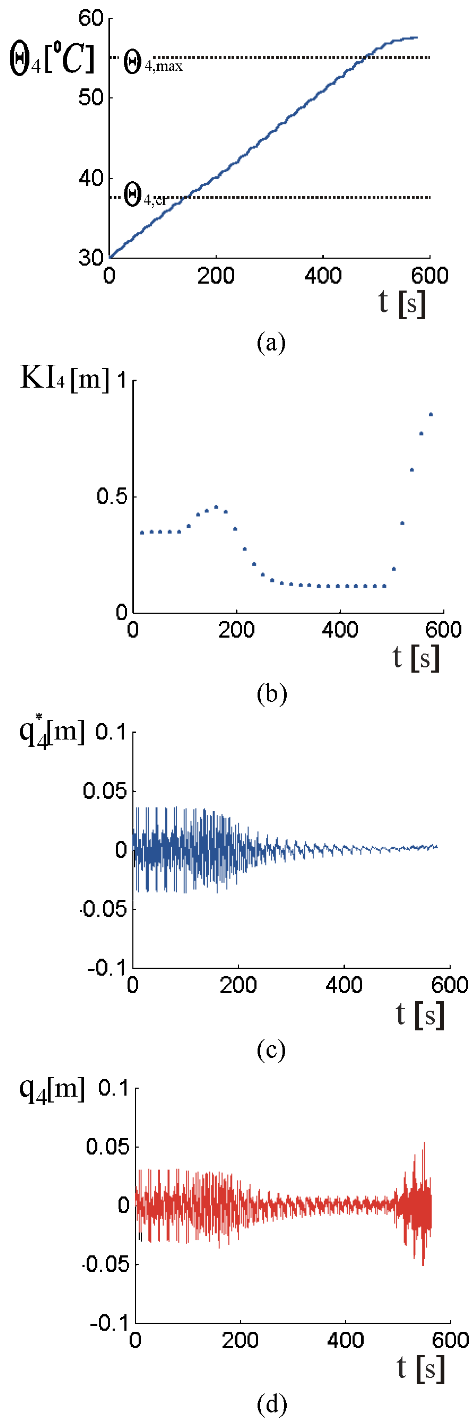

Fig. 7. Behavior of joint 4: (a) joint fatigue $\Theta_4(t)$, (b) joint involvement $KI_4(t)$, (c) reference motion $q_4^*(t)$, and (d) realized motion $q_4(t)$.

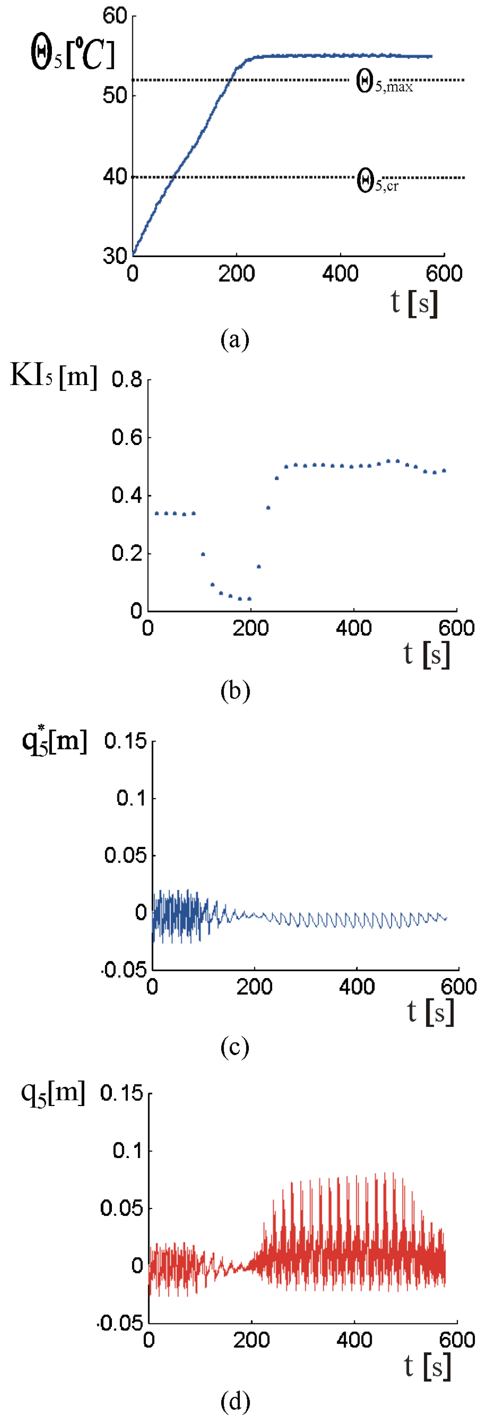

Fig. 8. Behavior of joint 5: (a) joint fatigue $\Theta_5(t)$, (b) joint involvement $KI_5(t)$, (c) reference motion $q_5^*(t)$, and (d) realized motion $q_5(t)$.

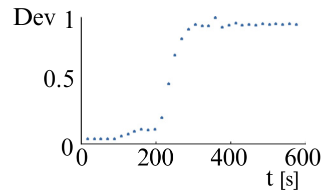

Fig. 9. Error in task execution: deviation (DEV) of realized letters from the reference (ideal) sequence.

starts to depress the engagement of joint 4, thus causing the drop of involvement KI_4, as is obvious from Fig. 7(b).

In spite of reconfiguration, the temperatures Θ_4 and Θ_5 continue to progress. This is due to the highly demanding task (relative to system parameters).

During phase 2, the task error DEV is slightly increased (Fig. 9). The small rise in writing error means that the tracking of the reference sequence is still good.

Phase 2 ends at about $t_2 = 190\,\mathrm{s}$, when the fatigue in joint 5 exceeds the next limit (upper level): $\Theta_5 \geq \Theta_{5,\max}$.

Phase 3 — Degeneration lasts for $t > t_2 \approx 190\,\mathrm{s}$. When joint 5 excedes $\Theta_{5,\max}$, phase 3 begins. The current limiter in the joint activates, reducing the joint drive. The reference joint motion [shown in Fig. 8(c)] still emerges from the inverse-kinematics calculation, expressing what the robot intends to do. The slightly increased magnitudes in the reference express the attempt of joint 5 to help joint 4 a bit [according to the simultaneous action of the two penalty functions (4) and (5)]. This means that the robot still intends to follow the reconfiguration procedure and do the job well, i.e. to write perfectly. However, the reduced joint drive will make joint 5 less controllable, and hence, the magnitude of realized motion in the joint will rise considerably [as seen in Fig. 8(d)]. So, tracking is not good any more. The kinematic involvement of joint 5 [KI_5 in Fig. 8(b)] will rise rapidly. However, one should note that this rise is not forced by a strong drive, on the contrary, it is caused by insufficient motor current and joint drive. The fatigue Θ_5 will stop rising and will reach the steady state [see Fig. 8(a)].

Joint 4, still strongly driven, will continue to track the reference motion [obvious from comparing Figs. 7(c) and (d)], and consequently, joint fatigue will continue to rise [Fig. 7(a)]. At about $t'' = 480\,\mathrm{s}$, joint fatigue exceeds the upper level: $\Theta_4 \geq \Theta_{4,\max}$. The current limiter in the joint activates and the drive reduction causes lower controllability. So, the joint will no longer track the reference, and oscillations will arise [compare Figs. 7(c) and (d)]. This increased kinematic involvement [obvious in Fig. 7(b) as well], caused by insufficient drive, will not contribute to motor heating. The reduced current will allow the temperature Θ_4 to reach the steady state [as shown in Fig. 7(a)].

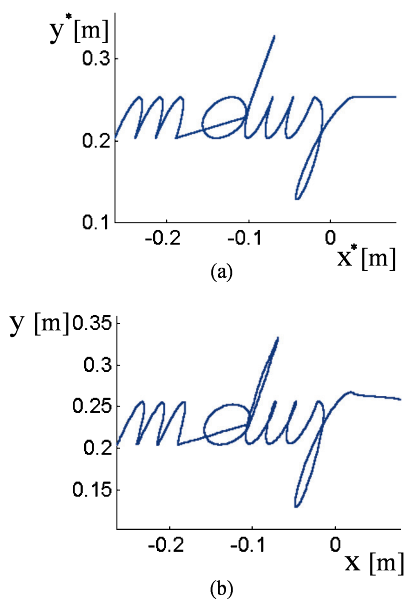

Fig. 10. Gradual degeneration of writing. (a) Reference sequence. Degenerated sequences are recorded for the following repetitions: (b) 12th forward sequence, time: $198 \leq t \leq 207$, (c) 14th forward sequence, time: $234 \leq t \leq 243$, (d) 22nd forward sequence, time: $378 \leq t \leq 387$.

During phase 3, the error in writing rapidly increases (see Fig. 9), which means that the quality of task execution becomes very low (that is why we talk about degeneration).

The deviation of actual letters from the reference pattern deserves more attention. Figure 10 shows how the realized letters gradually degenerate from the reference sequence. As mentioned above, during phases 1 and 2, the writing error was rather small. However, in phase 3, the trajectory rapidly degenerates. Figures 10(b)–(d) presents several realized sequences, all belonging to phase 3. Compared with the reference (ideal) sequence shown in Fig. 10(a); gradual degeneration is obvious. This is the handwriting of a tired robot.

6. Conclusion

A robotic arm engaged in handwriting was considered. The work was two-fold: to improve the knowledge in the biomechanics of handwriting, and to introduce some new concepts in robot control. The biomechanical principles humans apply when resolving kinematic redundancy were found and mathematically modeled in order to be applicable to robot control. This contributed to human-like motion of robots. Two approaches were proposed to model and control a human-like motion of a robot arm in a writing task.

The first approach, based on the concept of distributed positioning (DP), was suggested as a good model of arm motion in the phase where fatigue does not appear. The prescribed motion of the end-effector was distributed to a redundant number of arm joints in accordance with their acceleration capabilities. The justification of the usual inclination of letters was presented and the relation between the inclination, legibility, and finger involvement was discussed. It was found that for some prescribed level of legibility, an optimal inclination existed.

For the phase where fatigue appears, the concept of robot fatigue was proposed. It emulated the progress of biological fatigue. Penalty functions were utilized to ensure redistribution of the joint involvement when some of them "felt" fatigue. The arm automatically adapted to the situation, taking a new posture giving the exhausted joint the chance to rest while engaging the other joints more. The three phases of task execution, namely: *regular motion*, before the symptoms of fatigue; *reconfiguration*, after some joints feel fatigue; *degeneration*, caused by the excessively long, hard work that makes all joints tired, were discussed. The human-like reaction of a fatigued robot could be observed (thus being a kind of message), giving a chance to prevent undesirable consequences.

Appendix

The parameters used in the examples are given in the Tables 1–3. Table 1 presents the data about the robot arm. Table 2 defines the actuators and transmissions. Table 3 presents the parameters used in mathematical expressions simulated in the examples. Note that the values of the parameters were not chosen to be completely realistic but rather to stress and make more transparent the effects which we were elaborating.

Table 1. Arm parameters.

j	1	2	3	4	5
Length (m)	0.2	0.25	0.12	0.04	0.05
Mass (kg)	3.0	4.0	0.6	0.2	0.25

Table 2. Motors and transmissions.

j	1	2	3	4	5
Constant of torque equals the constant of back e.m.f	0.0239	0.0325	0.0214	0.0155	0.0155
Resistance R_j (Ω)	0.84	3.3	6.15	45.6	45.6
Transmission ratio: rotation-to-rotation (for $j = 1, 2, 3$), and rotation-to-linear (for $j = 4, 5$)	100	100	50	0.01	0.01

Table 3. Simulation parameters.

j	1	2	3	4	5
w_j	100	50	10	20	20
$k_{\varphi,j}$	3	3	3	300	900
K_{Pj}	1200	1200	1200	1500	1500
K_{Vj}	200	200	100	1200	1200
$\Theta_j(0)[^\circ\mathrm{C}]$	30	30	30	30	30
$\Theta_{j,cr}$	60	60	60	38	40
$\Theta_{j,\max}$	100	100	100	55	52
T_j	700	200	1100	1700	270
Z_j	200	100	250	10	50

References

1. T. Fukuda, R. Michelini, V. Potkonjak, S. Tzafestas, K. Valavanis and M. Vukobratovic, How far away is "Artificial man", in *IEEE Robotics and Automation Magazine*, March, 2001, pp. 66–73.
2. M. Vukobratovic and D. Juricic, Contribution to the synthesis of biped gait, in *IEEE Trans. Biomed. Eng.* **16**(2), 1–6 (1969).
3. D. Juricic and M. Vukobratovic, Mathematical modelling of a bipedal walking system, *ASME Publ. 72-WA/BHF-13*, 1972.
4. M. Vukobratovic, D. Hristic and Z. Stojiljkovic, Development of active anthropomorphic exoskeleton, *Med. Biol. Eng.* **12**(1), 66–80 (1974).
5. V. Potkonjak, M. Popovic, M. Lazarevic and J. Sinanovic, Redundancy problem in writing: From human to anthropomorphic robot arm, in *IEEE Trans. SMC Cybernetics*, Part B **28**(6), 790–805 (1998).
6. V. Potkonjak and A. Krstulovic, Mathematical modelling of a redundant antropomorphic arm, *Robotics and Autonomous Systems* **9**, 165–170 (1992).
7. V. Potkonjak and A. Krstulovic, Simulation of a redundant antropomorphic arm, *Robotics and Autonomous Systems* **9**, 171–179 (1992).
8. J. K. Salisbury and J. D. Abramowitz, Design and control of a redundant mechanisam for small motion, in *Proc. IEEE Int. Conf. Robotics and Automation*, St. Louis, MO, March 1985, pp. 323–328,
9. V. Potkonjak, Distributed positioning for redundant robotic systems, *Robotica* **8**(1), 61–67 (1990).
10. V. Potkonjak and A. Krstulovic, Contribution to the kinematics and dynamics of redundant robots via distributed positioning, *J. Intelligent and Robotic Systems* **5**, 229–239 (1992).
11. V. Potkonjak, G. Djordjevic, C. Milosavljevic, D. Antic and D. Popovic, Kinematic redundancy and sensor redundancy for enhancement of robot tracking performances, *J. Intelligent and Robotic Systems* **15**, 263–289 (1996).
12. V. Potkonjak, G. Djordjevic, C. Milosavljevic and D. Antic, Design of tactical and executive level of redundant robot control via distributed positioning, *Int. J. Robotic. and Autom.* **11**(3), 102–110 (1996).
13. V. Potkonjak, S. Tzafestas, D. Kostic and G. Djordjevic, Human-like behavior of robot arms: General considerations and the handwriting task "Mathematical Description of Human-Like Motion: Distributed Positioning and Virtual Fatigue", Part I, *Robotics and CIM* **17**, 305–315 (2001).

14. V. Potkonjak, D. Kostic, S. Tzafestas, M. Popovic, M. Lazarevic and G. Djordjevic, Human-like behavior of robot arms: General considerations and the handwriting task: "The Robot Arm in Handwriting", Part II, *Robotics and CIM* **17**, 317–327 (2001).
15. V. Potkonjak, D. Kostic, M. Rasic and G. Djordjevic, Motion in human and machine: A virtual fatigue approach, *IEEE Trans. SMC*: Systems and humans, Part A **32**(5), 582–595 (2002).
16. V. Potkonjak, S. Tzafestas and J. Radojicic, Modeling robot "psycho-physical" state and reactions — A new option in human-robot communication: "Concept and background", Part 2, *J. Intelligent and Robotic Systems* **35**(4), 339–352 (2002).
17. V. Potkonjak, S. Tzafestas, J. Radojicic and D. Kostic, Modeling robot "psycho-physical" state and reactions — A new option in human-robot communication: "Modeling and simulation", Part 2, *J. Intelligent and Robotic Systems* **35**(4), 353–364 (2002).
18. V. Potkonjak, S. Tzafestas, D. Kostic, G. Djordjevic and M. Rasic, Illustrating man-machine motion analogy in robotics — The handwriting problem, in *IEEE Robotics and Automation Magazine* **10**(1), 35–46 (2003).
19. L. Vodovnik and S. Rebersek, Improvements in voluntary control of paretic muscles due to electrical stimulation, *Electrical Stimulation as a Rehabilitation Method to Improve Abnormal Locomotion and Manipulation*, Final Report, Ljubljana, 1975, pp. 64–78.
20. Y. Giat, J. Mizrahi and M. Levy, A Model of fatigue and recovery in paraplegic's quadriceps muscle subjected to intermittent FES, *J. Biomech. Eng.* **118**, 357–366 (1996).
21. H. Cruse, E. Wischmeyer, M. Bruwer, P. Brockfield and A. Dress, On the cost functions for the control of the human arm movement, *Biol. Cybernet.* **62**, 519–528 (1990).
22. A. Liégeois, Automatic supervisory control of the configuration and behavior of multibody mechanisms, in *IEEE Trans. Systems, Man, and Cybernetics* **7**(12), 868–871 (1977).
23. T. F. Chan and R. V. Dubey, A weighted least-norm solution based scheme for avoiding joint limits for redundant joint manipulators, in *IEEE Trans. Robotics and Automation* **11**(2), 286–292 (1995).
24. D. E. Whitney, The mathematics of coordinated control of prosthetic arms and manipulators, *Trans. ASME, J. Dyn. Syst. Meas. Control* **94**(4), 303–309 (1972).

Veljko Potkonjak was born in Belgrade in 1951 and received his Dipl.Ing. in 1974, M.Sc. in 1977, and Ph.D. in 1981 in Electrical Engineering from the University of Belgrade, Serbia and Montenegro. He is currently a Professor in the Faculty of Electrical Engineering, University of Belgrade, where he teaches mechanics, robotics, biomechanics and cybernetics. He has also taught at the University of Nish, the Technical Faculty in Cacak, the National Technical University of Athens, and the American University of Athens. He has written three research monographs, contributed to several textbooks, and published more than 100 journal and conference papers.